Weighted Time Series Analysis for Electroencephalographic Source Localization

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Resumen

This paper presents a new method to estimate neural activity from electroencephalographic signals using a weighted time series analysis. The method considers a physiologically based linear model that take both spatial and temporal dynamics into account and a weighting stage to modify the assumptions of the model from observations. The calculated weighting matrix is included in the cost function used to solve the dynamic inverse problem, and therefore in the Kalman filter formulation. In this way, a weighted Kalman filtering approach is proposed including a preponderance matrix. The filter's performance (in terms of localization error) is analyzed for several SNRs. The optimal performance is achieved using the linear model with a weighting matrix computed by an inner product method.

Palabras clave: inverse problem, brain mapping, weighting matrix

1. INTRODUCTION

Electroencephalographic source reconstruction is a technique that estimate the sources of electrical currents (i.e. the current distribution) within the brain that give rise to recordable potential fields at the scalp. Estimation of the brain activity from electroencephalographic (EEG) measurements is known to be an ill-posed inverse problem (as there are an infinite number of different current sources that give rise to identical scalp recordings) that cannot be solved without some kind of regularization.

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Selection of dynamical models for neural activity as a constraint have improved the regularized solution.

However, the main restriction is the selection of the dynamical model [1]. Consequently, some variations of the dynamic model should be considered in order to improve the performance of the dynamic model for spatial and temporal behavior, such as more complex linear or nonlinear models, and temporal variability for the source and local neighbor interaction. Since the covariance is an assumption of the model in order to solve the inverse problem, the solution is highly dependent on that value. Therefore, it is necessary to improve the model from the observations and correct the considered [2].

In order to represent adequately the process dynamics, estimation of covariance from the process observation are used in [3] considering uniformly distributed variance for all current sources into the dynamic model. However, consider a uniform variance is an additional assumption that can increase the estimation error in the model since the neural activity is usually generated in a particular zone into the brain and therefore the variance is not uniform.

This article presents an EEG source reconstruction method that solves the inverse problem assuming a linear dynamic model. Additionally, a weighted Kalman filter approach is proposed where a weighting matrix is computed from the observation measurements in order to correct the model assumptions effectively representing the subjacent physiological phenomena. These models are applied over a realistic head structure. The analysis is made up from simulated EEG signals for different levels of noise.

2. MATERIALS AND METHODS

2.1. Inverse Problem Framework

The forward model for EEG data can be described by the discrete time measurement equation as follows:

$$\boldsymbol{\xi}^t = \boldsymbol{M}\boldsymbol{x}^t + \boldsymbol{\varepsilon}^t, \quad \boldsymbol{\varepsilon}^t \sim \mathcal{N}(\boldsymbol{0}, \sigma_\varepsilon \boldsymbol{I}) \tag{1}$$

where vector $\boldsymbol{x}^t = [x_1^t \cdots x_n^t \cdots x_N^t]^\top$, $\boldsymbol{x}^t \in \mathbb{R}^{3N \times 1}$, comprises the local $3-$dimensional current vectors $x_n^t \in \mathbb{R}^{3 \times 1}$, with $n = 1, 2, \ldots, N$, being N the number of distributed sources reflecting the brain activity. The so called lead field matrix, $\boldsymbol{M} \in \mathbb{R}^{d \times 3N}$, relates the current densities inside the brain $\boldsymbol{x}^t$ at time instant t with the EEG measurements, $\boldsymbol{\xi}^t \in \mathbb{R}^{d \times 1}$, and can be derived using the Maxwell equations for a specific head model [2]. The vector $\boldsymbol{\varepsilon}^t \in \mathbb{R}^{d \times 1}$ representing the measurement noise is modeled

as a (vector–valued) random variable by using both covariance matrix $\mathcal{E}\{(\varepsilon^t)(\varepsilon^t)^\top\} = \Sigma_\varepsilon = \sigma_\varepsilon \boldsymbol{I} \in \mathbb{R}^{d \times d}$ and cross-covariance $\mathcal{E}\{(\varepsilon^j)(\varepsilon^k)^\top\} = \boldsymbol{0}, \forall k \neq j$, where notation $\mathcal{E}\{\cdot\}$ stands for the expectation operator and $\boldsymbol{I} \in \mathbb{R}^{d \times d}$ stands for the Identity matrix and $\mathcal{N}(,)$ denotes the normal distribution.

Generally, (1) represents an inverse problem and the estimation $\widetilde{\boldsymbol{x}}^t$ can be obtained by minimizing the objective function given for each time t independently, as follows:

$$\widetilde{\boldsymbol{x}}^t = \min_{\boldsymbol{x}^t}\left\{\left\|\boldsymbol{\xi}^t - \boldsymbol{M}\boldsymbol{x}^t\right\|^2\right\} \tag{2}$$

being $\|\cdot\|^2$ is the $l-2$ norm. However, the measured EEG $\boldsymbol{\xi}^t$ comes from a process where the activity is not uniform and therefore its associate variance defined as $\boldsymbol{\Sigma_\xi} \in \mathbb{R}^{d \times d}$ is not uniform over all d channels. It is usually considered gaussian, with zero mean and a variance inversely proportional to the signal magnitude [4]. In order to account for inhomogeneous variance, the following weighted least squares minimization problem can be considered

$$\widetilde{\boldsymbol{x}}^t = \min_{\boldsymbol{x}^t}\left\{\left\|\boldsymbol{W}\left(\boldsymbol{\xi}^t - \boldsymbol{M}\boldsymbol{x}^t\right)\right\|^2\right\} \tag{3}$$

where $\boldsymbol{W} \in \mathbb{R}^{d \times d}$ is a weighting matrix, which is diagonal in the case of uncorrelated gaussian noise.

Since equation (3) is an ill-posed inverse problem, an improvement in the solution can be obtained through regularization methods by including prior information (spatial and temporal constraints) and can be formulated as follows:

$$\widetilde{\boldsymbol{x}}^t = \min_{\boldsymbol{x}^t}\left\{\left\|\boldsymbol{W}\left(\boldsymbol{\xi}^t - \boldsymbol{M}\boldsymbol{x}^t\right)\right\|^2\right. \tag{4}$$

$$\left. + \gamma^2\left\|\boldsymbol{W_s}\left(\boldsymbol{x}^t - \boldsymbol{f}\left(\boldsymbol{x}^{t-1}, \boldsymbol{x}^{t-2}\right)\right)\right\|\right\} \tag{5}$$

where $\boldsymbol{W_s}$ is an operator associated to the spatial constraint and γ is the regularization hyper parameter [2]. $\boldsymbol{f}(,)$ is a vectorial function that describes the second order dynamics of the current density (and thus the neuronal activity) and represent the state evolution equation is assumed to be a second order linear model defined as

$$\boldsymbol{x}^t = \boldsymbol{f}(,) + \boldsymbol{\eta}^t = \boldsymbol{\Phi}_1\boldsymbol{x}^{t-1} + \boldsymbol{\Phi}_2\boldsymbol{x}^{t-2} + \boldsymbol{\eta}^t, \quad \boldsymbol{\eta}^t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma_\eta}), \tag{6}$$

where $\boldsymbol{\eta}^t \in \mathbb{R}^{3N \times 1}$ is the so–called process noise representing the external stimulus, and the matrix $\boldsymbol{\Sigma_\eta} \in \mathbb{R}^{3N \times 3N}$ denotes the covariance matrix of

$\boldsymbol{\eta}^t$, i.e., $\boldsymbol{\mathcal{E}}\{(\boldsymbol{\eta}^k)(\boldsymbol{\eta}^k)^\top\} = \boldsymbol{\Sigma_\eta}$. Furthermore, the random variables $\boldsymbol{\eta}^j$ and $\boldsymbol{\eta}^k$ with $k \neq j$ are supposed to be uncorrelated, i.e., $\boldsymbol{\mathcal{E}}\{(\boldsymbol{\eta}^j)(\boldsymbol{\eta}^k)^\top\} = \mathbf{0}, \forall\, k \neq j$, and where $\boldsymbol{\Phi}_1 = \varphi_1\boldsymbol{I} + \psi_1\boldsymbol{L}$ and $\boldsymbol{\Phi}_2 = \varphi_2\boldsymbol{I}$; with $\boldsymbol{\Phi}_i \in \mathbb{R}^{3N\times3N}$, $i = 1,\ldots,2$, being $\boldsymbol{I} \in \mathbb{R}^{3N\times3N}$ the identity matrix. Notation $\boldsymbol{L} \in \mathbb{R}^{3N\times3N}$ stands for the matrix operator that represents the spatial interaction among sources [3]. Equation (4) represents the solution of a dynamic inverse problem that can be solved by application of Kalman filtering methods. For that purpose, equation (6) can be reformulated in the form of a first order augmented model as follows:

$$\underbrace{\begin{bmatrix} \boldsymbol{x}^t \\ \boldsymbol{x}^{t-1} \end{bmatrix}}_{\boldsymbol{z}^t} = \underbrace{\begin{bmatrix} \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 \\ \boldsymbol{I} & \mathbf{0} \end{bmatrix}}_{\boldsymbol{\Gamma}} \begin{bmatrix} \boldsymbol{x}^{t-1} \\ \boldsymbol{x}^{t-2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\eta}^t \tag{7}$$

2.2. Weighting preprocessing

Typically, since $\boldsymbol{W}$ is associated with the correlation matrix of an EEG time series $\boldsymbol{\xi}^t$ at time $t \in T$, it can be defined as a positive definite weighting matrix $\boldsymbol{W} \in \mathbb{R}^{d\times d}$ that distinguishes channels, effectively representing the subjacent physiological phenomena, and according to some evaluation measure[5]. Matrix $\boldsymbol{W}$ is also called a multivariate projection matrix and is defined, in the case of uncorrelated Gaussian noise, as follows

$$\boldsymbol{W} = \operatorname{diag}(\boldsymbol{w}) \tag{8}$$

being $\boldsymbol{w} = [w_1\,w_2\,\ldots w_d]$, $\boldsymbol{w} \in \mathbb{R}^{d\times 1}$ a weighting vector, where $w_i \geq 0$ is the $i - th$ weight. This diagonal preponderance matrix improves the model assumptions of (1) in the solution of the dynamic inverse problem. Therefore, the estimated activity resulting from the inverse problem solution including the weighting matrix are modified correcting the covariance matrix of (1).

Since $\boldsymbol{W}$ consider the variability of the EEG time series $\boldsymbol{\xi}^t$, it can be calculated from a matrix $\boldsymbol{H} \in \mathbb{R}^{d\times T}$ constructed as follows:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{\xi}^T & \boldsymbol{\xi}^{T-1} & \cdots & \boldsymbol{\xi}^1 \end{bmatrix}^\top \tag{9}$$

where T is the total number of samples.

Matrix $\boldsymbol{W}$ is estimated according to the following optimization problem

$$\min_{\boldsymbol{U}} \{\left\| \boldsymbol{H} - \widetilde{\boldsymbol{H}} \right\|^2\} = \sum_{j=q+1}^{p} \lambda_j \tag{10}$$

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where $\widetilde{\boldsymbol{H}} \in \mathbb{R}^{d \times T}$ is the reconstruction matrix in a low dimensional space so that $\widetilde{\boldsymbol{H}} = \boldsymbol{ZV}^{\mathsf{T}}$, matrix $\boldsymbol{Z} = \boldsymbol{HV} \in \mathbb{R}^{d \times q}$ and $\boldsymbol{V} \in \mathbb{R}^{T \times q}$ is the orthogonal projection matrix corresponding to the eigenvectors of matrix $\boldsymbol{H}^{\mathsf{T}}\boldsymbol{H}$, λ_j is the j-th eigenvalue of the same matrix, and $\|\cdot\|^2$ represents the squared inner norm. Therefore, from the previous optimization problem can be accomplished the following weighting vector:

$$\boldsymbol{\rho} = \sum_{j=1}^{q} \lambda_j (\boldsymbol{v}_j).^2 \tag{11}$$

where $\boldsymbol{v}_j$ is the j-th column of $\boldsymbol{V}$, and $(\boldsymbol{v}_j).^2$ stands for each element squared. Then, corresponding weighting matrix is $\boldsymbol{W} = \mathrm{diag}(\sqrt{\boldsymbol{\rho}})$.

2.3. Weighted Kalman filter

Equation (4) can be solved in the Kalman filter framework, where the time update equations are defined as follows:

$$\widehat{\boldsymbol{z}}^{t-} = \boldsymbol{\Gamma}\widehat{\boldsymbol{z}}^{t-1} \tag{12a}$$

$$\boldsymbol{\Sigma}^{t-} = \boldsymbol{\Gamma}\boldsymbol{\Sigma}^{t-1}\boldsymbol{\Gamma}^{\mathsf{T}} + \boldsymbol{\Sigma_\eta} \tag{12b}$$

where $\widehat{\boldsymbol{z}}^{t-}$ is defined as a priori estimation of $\widehat{\boldsymbol{z}}^t$. Then, we compute the measurement update equations for the state filter described by

$$\boldsymbol{G}^t = \boldsymbol{\Sigma}^{t-}\boldsymbol{M}^{\mathsf{T}}\boldsymbol{W}^{\mathsf{T}} \left(\boldsymbol{WM\Sigma}^{k-}\boldsymbol{M}^{\mathsf{T}}\boldsymbol{W}^{\mathsf{T}} + \boldsymbol{\Sigma}_\varepsilon \right)^{-1} \tag{13a}$$

$$\widehat{\boldsymbol{z}}^t = \widehat{\boldsymbol{z}}^{t-} + \boldsymbol{G}^t\boldsymbol{W} \left(\boldsymbol{\xi}^t - \boldsymbol{M}\widehat{\boldsymbol{z}}^{t-} \right) \tag{13b}$$

$$\boldsymbol{\Sigma}^t = \left(\boldsymbol{I} - \boldsymbol{G}^t\boldsymbol{WM} \right) \boldsymbol{\Sigma}^{t-} \tag{13c}$$

3. RESULTS AND DISCUSSIONS

3.1. Simulated EEG Recordings

Initially, system dynamics is approximated through linear time invariant model according to [6] considering anatomic constraints related with spatial coupling between sources. Testing is carried out for simulated EEG recordings. In this way, a localization error e, according to [2] is used for each trial:

$$e = \boldsymbol{\mathcal{E}}\{d_t : \forall t \in T\}, \tag{14}$$

where $d_t = |\boldsymbol{r}_{dip_t} - \boldsymbol{r}_{dip}|$ is the distance from the $n - th$ element position $\boldsymbol{r}_{dip_t}$ to the true source position $\boldsymbol{r}_{dip}$ and T is the total number of samples.

A total localization error is computed as a mean value of the errors for each trial, and error bars are the sample standard deviations (from the sample of 100 trials). At each source, the 3D local current density vector is mapped, as usual, to the 19 electrode sites for the 10-20 system. A realistic three layer head model is used for the solution [3].

The evaluation of the inverse solution is achieved for simulated EEG data where underlying sources are known. Here, the temporal dynamics are suggested to be simulated using the linear model given by (6). The parameters used for simulation are $\varphi_1 = 1{,}2$, $\psi_1 = 0{,}05$, and $\varphi_2 = -0{,}9$, and the activity in the source is modeled as a combination of three sine functions whose frequencies are evenly spaced in the alpha band $(8 - 12$ Hz$)$, since the clinical data used in real EEG recordings display prominent alpha activity [3]. Specifically, the simulated brain dynamics are generated for 1000 time points, assuming a sampling rate of 1000 Hz. Besides, the 100 repetitions of one second of synthetic EEG data are generated from the simulated current densities by multiplication with the lead field matrix $\boldsymbol{M}$, taking into account the additive noise with six different signal noise ratios (SNR): 1 dB, 5 dB, 10 dB, 15 dB, 20 dB and 25 dB.

3.2. Weighting Analysis

The following weighting matrices are considered

- $\boldsymbol{W} = \boldsymbol{I}$, being $\boldsymbol{I} \in \mathbb{R}^{d \times d}$ the identity matrix, and

- $\boldsymbol{W} = \mathrm{diag}(\sqrt{\boldsymbol{\rho}})$.

In order to consider the effect of the noise level in the estimation of the weighting matrix $\boldsymbol{W}$, an estimation of the weights for several SNR's is performed. This analysis is performed for the case of matrix $\boldsymbol{W} = \mathrm{diag}(\sqrt{\boldsymbol{\rho}})$ with and without normalization. In Fig. 1(a) it is shown the preprocessing weights for several noise conditions without normalization. It is shown for the case of normalization that the variance increase its values when it is not normalized in spite of the noise levels.

However, as shown in Fig. 1(b) it is clear than the matrix values are stable for the several levels of noise in ρ method. In Fig. 2 is depicted the simulated and estimated neural activity for several weighting matrices at $t = 0$, $t = 0{,}1$ and $t = 0{,}2$ seconds. As shown in Fig. 2, when the weighted Kalman filter is applied, an improvement on the estimation is achieved.

The estimation error in case of a linear model is computed using the selected weighting matrices, in case of a time invariant linear model. In Fig. 3 it is shown the estimation results using the Kalman filtering for state estimation.
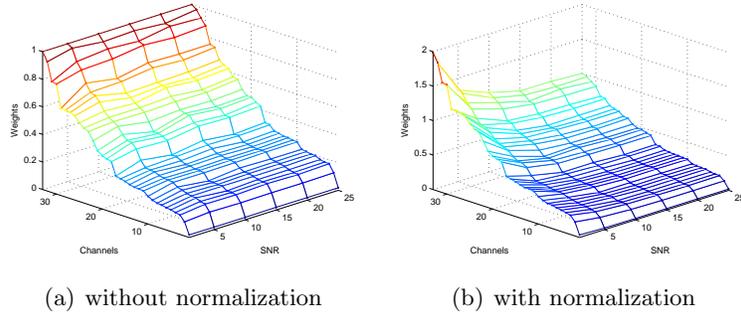
(a) without normalization   (b) with normalization

Figura 1: Preprocessing weights $\text{diag}(\sqrt{\boldsymbol{\rho}})$

$\boldsymbol{W} = \text{diag}(\sqrt{\boldsymbol{\rho}})$ True $\boldsymbol{W} = \boldsymbol{I}$
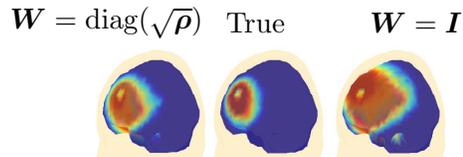
Figura 2: Estimated activity at time instant $t = 0{,}2$ for several weighting matrices

4.   CONCLUSIONS AND FUTURE WORK

In this paper, we have addressed the dynamical inverse problem of EEG source reconstruction, using a new method to improve the model through a weighted approach. The obtained results demonstrate that the models with including weighting matrices perform better than the assumed model. This improvement is due to the fact that the model with weighting matrices corrects the initial assumption from observations. These results are confirmed in this paper for simulated signals over several SNR values where the weighted model reached the best performance. In future work, time varying weighting matrix could be studied in order to further improve the performance of the estimator.
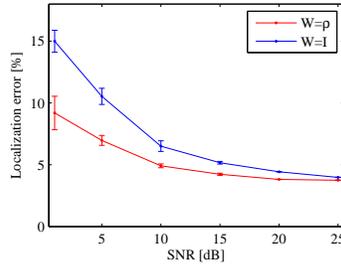
5.   ACKNOWLEDGMENTS

Figura 3: Estimation error for different weights

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